Is there a Gravitational Effect which is analogous to Electrodynamic Induction?[1]

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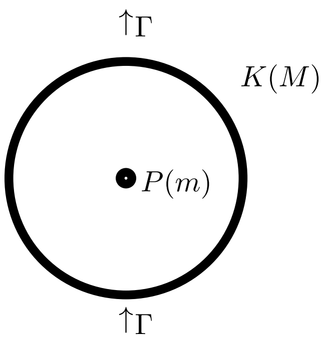
The question posed in the title can be formulated, in a concise special case, as follows. Consider a system of ponderable masses, composed of a spherical shell K with mass M uniformly distributed over the surface, and at the center of this spherical shell the indicated material point P with mass m. Does a force act on the fixed point P if I impart an acceleration Γ to the shell K? The following arguments will lead us to the view that such a force is to be found and to a first approximation of its value.



1. From Relativity Theory the inertial mass of a closed physical system depends on its energy content in a manner such that an increase in energy of the system by E results in an increase of the inertial mass by $\frac{E}{c^2}$, where c is the speed of light in a vacuum. We designate the inertial mass of K by M in the absence of P, and the inertial mass of P by m in the absence of K. Or, in other words, with $M + m$ the inertial mass of the combined system P and K in the case where m is located infinitely distant from K, it then follows that the inertial mass of the system K and m in the case where m is located at the center of K, is

$$M + m - \frac{kMm}{Rc^2} \tag{1}$$

where k is the gravitational constant and R is the radius of K. Then (at least as a first approximation) $\frac{kMm}{R}$ is the energy that must be expended to move P from the center of K to infinity.

2. In a work that will appear soon in the Annalen der Physik, I have, based on a certain hypothesis on the nature of static gravitational fields which I have developed, found that a material point in a static gravitational field moves according to the following equation:

$$\frac{d}{dt}\left\{\frac{\frac{\dot{x}}{c}}{\sqrt{1-\frac{q^2}{c^2}}}\right\} = \frac{-\frac{dc}{dx}}{\sqrt{1-\frac{q^2}{c^2}}} + \frac{R_x}{m} \quad \text{and so on.}$$

Here we set $\dot{x} = \frac{dx}{dt}$, and q is the velocity of the material point, m its mass, R_x is the force acting on it, c is the speed of light, which is to be viewed as a function of the coordinates (x, y, z). From this equation, among other things, we get $\frac{mc}{\sqrt{1-\frac{q^2}{c^2}}}$ for

[1] Translation of "Gibt es eine Gravitationswirkung, die der electrodynamischen Inductionswirkung analog ist?," *Vierteljahrsschrift für gerichtliche Medizin und öffentliches Sanitätswesen* **(44)** p. 37–40. 1912. Translation by M. D. Godfrey(email: michaeldgodfrey@gmail.com)

the energy of the material point and, to a first approximation, $\frac{m}{2}\frac{q^2}{c}$ is seen to be the corresponding kinetic energy. To obtain the kinetic energy in the usual units, one must multiply this result by the constant c_0, which is the speed of light at infinity; this last is equal to setting the speed of light to that of our gravitational potential which holds in the middle. The kinetic energy L is therefore in the usual units

$$L = \frac{m}{2} q^2 \frac{c_0}{c}$$

Therefore in order that the expression for L may be known for a specified location, we have to determine the value of c as a function of $x\ y\ z$. From the given equation of motion for a sufficiently slowly moving point, such that it does not act on the gravity field:

$$\ddot{x} = c\frac{dc}{dx} \quad \text{and so on.}$$

or, when one defines the gravitational potential Φ in a similar way:

$$\frac{d\Phi}{dx} = c\frac{dc}{dx} \quad \text{and so on.}$$

From this by integration, if we define Φ_0 to be the gravity potential at infinity, with sufficient accuracy:

$$\Phi_0 - \Phi = c_0(c_0 - c) = c_0^2(1 - \frac{c}{c_0})$$

$$\text{or} \quad \frac{c}{c_0} = 1 - \frac{\Phi_0 - \Phi}{c_0^2}.$$

For a material point in the interior of K, $\Phi_0 - \Phi$ is equal to $\frac{kM}{R}$, so that in its neighborhood it holds that:

$$L_p = \frac{m}{2} q^2 \left(1 + \frac{kM}{Rc_0^2}\right),$$

therefore for an inertial mass m' moving due to K

$$m' = m + \frac{kM}{Rc_0^2} \tag{2}$$

This is a result of great interest. It shows that the presence of the inertial shell K within which is found the material point P increases the inertial mass of P. This fact suggests the conjecture that the entire inertia of a material point is an effect of the presence of all the other masses, based on a kind of interaction with the latter.[1] The extent to which this view is correct will be seen when we come to be in the happy position of having an effective gravitational dynamics.

[1] It is this very position, which E. Mach in his acute research argued on the subject. (E. Mach, Die Entwicklung der Principen der Dynamik. Zweites Kapital. VI. Newtons Ansichten über Zeit, Raum und Bewegung.)

It is clear, that in the same way the inertial mass of K will be increased through the effect of P. Through a similar argument quite analogous considerations one reaches from the effect of P influenced by the inertial mass M' of K

$$M' = M + \frac{kmM}{Rc_0^2} \tag{3}$$

3. We now ask about the force F or f, which are necessary in order to reach a definite determination of the masses M or m effect the acceleration of Γ or γ. Define A, a, and α as unknown coefficients, and so in any case setting:

$$\left\{\begin{array}{c} F = A\Gamma + \alpha\gamma \\ f = a\gamma + \alpha\Gamma \end{array}\right\} \tag{4}$$

The coefficients of the second equation (a) are chosen the same in both equations, because the reaction from K on P, in case only K is moved, obviously both must be of the same size, as the reaction of P on K, in case only P is moved.

The coefficients A, a and α will become apparent from a consideration of the three special cases, which are shown in equations (1), (2), and (3).

In the first case K and P are both changed. The common acceleration is γ. From (4) and (1) one gets

$$F + f = (A + a + 2\alpha)\gamma = \left(M + m - \frac{kMm}{Rc^2}\right)\gamma$$

or

$$A + a + 2\alpha = M + m - \frac{Mkm}{Rc^2} \tag{1a}$$

In the second case, in which P alone is moved, one has from the two equations (4) and (2)

$$f = a\gamma = \left(m + \frac{kmM}{Rc^2}\right)\gamma$$

or

$$a = m + \frac{kmM}{Rc^2} \tag{2a}$$

The third case analogously provides

$$A = M + \frac{kmM}{Rc^2} \tag{3a}$$

From equations (1a), (2a) and (3a) it follows that

$$\alpha = -\frac{3}{2}\frac{kMn}{Rc^2}$$

For the case where only K varies, and P held fixed, the second equation of (4) will be used to find the value of α in

$$(-k) = \frac{3}{2}\frac{kmM}{Rc^2}\Gamma$$

k is thus the force that must be applied to the material point P in order for it to come to rest, thus $(-k)$ is the force exerted by the acceleration Γ with the spherical shell P force. This has the same effect as the acceleration – in contrast to the corresponding interaction between equal electrical masses.